Computation of Matrix Rank by Linear Solves

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Abstract

Parameter - dependent matrices arise in many applications and notably in continuation and bifurcation problems. Detecting rank changes and computing parameter values for which a matrix has a prescribed (low) rank deficiency is a fundamental task. Since all types of matrices (dense, sparse, structured, ...) can arise in this context, it is desirable that the methods should be as general as possible (SVD and QR are ruled out for reasons of storage if A is sparse).

We present an approach that uses only linear solves with the matrix itself and some bordered extensions with small borderwidth and randomly generated border elements. This approach is closely related to the Ljapunov-Schmidt reduction of a nonlinear equation where the numerical implementation precisely uses bordered matrices. We give extensive numerical evidence and discuss the influence of the randomness in the choice of the borders.

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